Growth and Decay

# **Exponential Functions**

A <u>exponential function</u> is a function where the variable x, is found in the exponent  $(y = 2^x)$ . The standard form of exponential functions is  $y = ab^x$ . Since the variable is found in the exponent, these graphs are no longer straight lines. In a straight line, the "rate of change" is the same across the graph, but in these graphs the "rate of change" will either increase or decrease across the graphs.

## **Exponential Growth and Decay**

# Watch how the graphs of these functions change based upon the values of a and b



Many real world phenomena can be modeled by functions that describe how things grow or decay as time passes. Examples of such phenomena include the studies of populations, bacteria, the AIDS virus, radioactive substances, electricity, temperatures and credit payments, to mention a few.

Any quantity that grows or decays by a fixed percent at regular intervals is said to possess **exponential growth** or **exponential decay**.

*Exercise* **#1**: Evaluating Functions.

1) Evaluate the function  $f(x) = 2^x$ for x = 5 2) Evaluate the function  $g(x) = \frac{1^x}{3} + 5$ for x = -2

At the Algebra level, there are two functions that can be easily used to illustrate the concepts of growth or decay in applied situations. When a quantity grows by a fixed percent at regular intervals, the pattern can be represented by the functions,

Growth	Decay
y = a(1 +	$y = a(1-r)^x$
a r x	<ul> <li>initial amount before measuring growth/decay</li> <li>growth/decay rate (often a percent and must be converted to a decimal)</li> <li>number of time intervals that have passed</li> </ul>

*Exercise* **#2**: In 1995, there were 285 cell phone subscribers in the small town of Centerville. The number of subscribers **increased** by 45% per year after 1995. How many cell phone subscribers were in Centerville in 2006?

*Exercise* #3: Kathy plans to purchase a car that **depreciates** (loses value) at a rate of 14% per year. The initial cost of the car is \$21,000. What will the value of the car be after 3 years?

*Exercise* **#4**: Daniel's Print Shop purchased a new printer for \$35,000. Each year it **depreciates** at a rate of 5%. What will its approximate value be at the end of the fourth year?

**Exercise #5:** The breakdown of a sample of a chemical compound is represented by the function  $p(t) = 300(0.5)^t$  where p(t) represents the number of milligrams of the substance and t represents the time, in years. In the function p(t), explain what 0.5 and 300 represent.

**Exercise** #6: Kristen invests money in an account as represented by the function  $A(x) = 1000(1.03)^t$  where A(x) represents the amount of money in her account and t represents the time, in years. In the function A(x), explain what the 1000 and 1.03 represent.

*Exercise* #7: A bank is advertising that new customers can open a savings account with a  $3\frac{3}{4}$ % interest rate compounded annually. Robert invests \$5,000 in an account at this rate. If he makes no additional deposits or withdrawals on his account, find the amount of money he will have, to the nearest cent, after three years.

8) Mr. Smith invested \$2,500 in a savings account that earns 3% interest compounded annually. He made no additional deposits or withdrawals. Which expression can be used to determine the number of dollars in this account at the end of 4 years?

(1)  $2500(1 + 0.03)^4$  (2)  $2500(1 + 0.3)^4$ (3)  $2500(1 + 0.04)^3$  (4)  $2500(1 + 0.4)^3$ 

9) The value, y, of a \$15,000 investment over x years is represented by the equation

 $y = 15000(1.2)^{\frac{x}{3}}$ . What is the *profit* (interest) on a 6-year investment?

- (1) \$6,600 (2) \$10,799
- (3) \$21,600 (4) \$25,799

10) The value of a car purchased for \$20,000 **decreases** at a rate of 12% per year. What will the value of the car be after 3 years? (1) \$12,800,00 (2) \$13,629,44

(1) \$12,800.00	(2) \$13,629.44
(3) \$17,600.00	(4) \$28, 098.56

\_\_\_\_ 11) The current population of a town is 10,000. If the population, P, **increases** by 20% each year, which equation could be used to find the population after t years?

Name	Date	Period
Algebra	Exponential	9C HW

# **Exponential Functions Homework**

1) A mouse population starts with 2,000 mice and grows at a rate of 5% per year. The number of mice after t years can be modeled by the equation P(t) = 2000(1.05)<sup>t</sup>. What is the average rate of change in the number of mice between the second year and the fifth year, rounded to the nearest whole number?

(1) 116	(2) 348
(3) 2205	(4)2553

- 2) Which situation could be modeled with an exponential function?
  - (1) the amount of money in a savings account where \$150 is deducted every month
  - (2) the amount of money is Suzy's piggy bank which she adds \$10 to each week
  - (3) the amount of money in a certificate of deposit that gets 4% interest each year
  - (4) the amount of money in Jaclyn's wallet which increases and decreases by different amounts

#### \_\_\_\_\_ 3) Which function models the relationship shown in the table?

(1)	$f(x) = \frac{1}{2}$	$\frac{100}{x}$
(2)	f(x) =	$100\left(\frac{1}{2}\right)^x$
(3)	f(x) =	$50(2)^{x}$
(4)	f(x) =	$200\left(\frac{1}{2}\right)^{x}$

x	f(x)
1	100
2	50
3	25
4	12.5
5	6.25

- 4) Cassandra bought an antique dresser for \$500. If the value of her dresser increases 6% annually, what will be the value of Cassandra's dresser at the end of 3 years to the nearest dollar?
  - (1) \$415(2) \$590(3) \$596(4) \$770

\_ 5) The current student population of the Brentwood Student Center is 2,000. The enrollment at the center increases at a rate of 4% each year. To the nearest whole number, what will the student population be closest to in 3 years'?

(1) 2,240	(2) 2,250
(3) 5,488	(4) 6,240

- \_ 6) If the function f(x) represents the number of words that Janet can type in x minutes, what is the possible domain for the function?
  - (1) the set of integers (2) the set of non-negative integers
  - (3) the set of real numbers (4) the set of irrational numbers
- \_\_\_\_\_ 7) Labor at a car repair shop can be represented by the function:

 $total charges for repairs = \begin{cases} 150, \ 0 < h \le 1\\ 150 + 80(h-1), \ h > 1 \end{cases}$ If h represents the number of hours worked, what is the charge for a 3 hour car repair? (1) \$150 (2) \$230 (3) \$310 (4) \$390

8) Find the average rate of change of the function shown to the right that represents the amount of money in a savings account in Lender's Bank.

Week	Balance
1	\$128
2	\$142
3	\$156
4	\$170
5	\$184

9) The amount of money A accrued at the end of n years when a certain amount P is invested at a compound annual rate r is given by  $A = P(1 + r)^n$  If a person invests \$150 at 5% interest compounded annually, find the approximate amount obtained at the end of 5 years.

10) Find the amount accumulated to the nearest cent on \$700 compounded annually for 3 years at 9%.

11) Suppose that in 1980, one of your ancestors invested \$46 compounded annually at 6.5%. If this money were left to you, how much would you have had at the end of 1997? Round to the nearest dollar

12) A position at a local company has a starting salary of \$15,000. The salary is expected to increase by 5% each year. What will the salary be after 5 years

#### Review Section:

- 13.) The table shows the average yearly balance in a savings account where interest is compounded annually. No money is deposited or withdrawn after the initial amount is deposited. Which type of function best models the given data?
  - [1] linear function with negative rate of change
  - [2] linear function with positive rate of change
  - [3] exponential decay
  - [4] exponential growth

Year	Balance, in Dollars
0	380.00
10	562.49
20	832.63
30	1232.49
40	1824.39
50	2700.54

# 14.) Draw the graph of $y = \sqrt{x} - 1$ on the set of axes below. Don't forget your table!!



Name	Homewo	rk Answers		Date	Period
Algebra				Exponential	9C HW
1.) [1]					
2.) [3]					
3.) [4]					
4.) [3]					
5.) [2]					
6.) [3]					
7.) [3]					
8.) m = 14	1	9.) \$191.44	10.) \$906.52	11.) \$134	
12.) \$19,	144.22	13.) [4]	14.) Table & Graph		

# **Exponential Functions (Day 2)**

A <u>exponential function</u> is a function where the variable x, is found in the exponent  $(y = 2^x)$ . The standard form of exponential functions is  $y = ab^x$ . Since the variable is found in the exponent, these graphs are no longer straight lines. In a straight line, the "rate of change" is the same across the graph, but in these graphs the "rate of change" will either increase or decrease across the graphs.

#### Exercise #1:

- If the savings in a bank account can be modeled by the function  $S(t) = 250(1.045)^t$ . Which of the following is true?
  - (1) The initial amount deposited was \$250 and the interest earned is 45%.
  - (2) The initial amount deposited was \$2.50 and the interest rate is 4.5%.
  - (3) The initial amount deposited was \$250 and the interest rate is 4.5%.
  - (4) The initial amount deposited was \$2.50 and the interest rate is 45%.

#### Exercise #2:

Which of the following equations would model an exponential quantity that begins at a level of 16 and decreases at a constant rate of 8% per hour?

(1)  $Q = 16(0.92)^t$  (3)  $Q = 16(1.08)^t$ (2)  $Q = 16 + 0.92^t$  (4)  $Q = 16(-7)^t$ 

#### Exercise #3:

\_\_\_\_ If \$350 is placed in a savings account that earns 3.5% interest applied once a year, then how much would the savings account be worth after 10 years?

(1) \$522.88	(3) \$472.50
(2) \$426.34	(4) \$493.71

## *Exercise* #4: Use your knowledge of exponential functions to match the rule and graph.









*Exercise* #5: Use your knowledge of exponential functions to identify the domain and range.

A)	Domain:	
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B) Domain: \_\_\_\_\_

Range:		
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Range:	
- 0-	 _





C) Domain: \_\_\_\_\_

Range: \_\_\_\_\_



D) Domain: \_\_\_\_\_

Range: \_\_\_\_\_



Name	 	
Algebra		

Exponential

9D HW

Growth and Decay

**Exponential Functions (Day 2) Homework** 

- 1) Identify the rule for the function graphed to the right. (1)  $f(x) = -4(2)^x$ 
  - (2)  $f(x) = 4(2)^x$ (3)  $f(x) = -4\left(\frac{1}{2}\right)^{x}$ (4)  $f(x) = 4\left(\frac{1}{2}\right)^{x}$
- 2) If  $g(x) = x^2 + 3x$ , what is the value of g(-3)? (1) 0(2)3(3) 18 (4) 21

3) Which function below will result in a downward vertical shift of the graph of the parent function  $y = x^2$ ? (1)  $y = \frac{1}{2}x^2$ 

(2)  $y = 2x^2$ (4)  $y = x^2 - 1$ (3)  $y = x^2 + 2$ 

\_\_\_\_4) The graph of the equation  $y = 3^x$  contains which point? (2)  $\left(-2, \frac{1}{9}\right)$ (4)  $\left(-3, -\frac{1}{9}\right)$ (1) (1,9) (3) (2,6)

5) Find the average rate of change between f(-4) and f(-1) in the function  $f(x) = x^2 + 2x - 8$ (1) - 9(2) - 3(3) 3 (4) 9



- 6) Joey's taxi charges \$10 for the initial service of any drive. Then, the fee for each mile is \$0.75. Which type of function is represented by this situation?
  - (1) linear (2) exponential
  - (3) quadratic (4) absolute value

\_\_\_\_7) Which relation is a function?

- $(1) \{(0,-2), (4,10), (-1,-5), (2,4)\}$
- $(2) \{ (2,3), (2,5), (2,7), (2,9) \}$
- $(3) \{(4,8), (2,-3), (1,1), (2,-1)\}$
- $(4) \{(0,0), (0,3), (3,1), (4,-1)\}$

8) Which graph matches the equation  $y = -4(2)^{x}$ ?



9) A photocopier is purchased for \$5,200 and depreciates in value by 15% per year. Which equation best describes the value of the photocopier in t years?

(1)  $y = 5200(0.15)^x$ 

- (2)  $y = 5200(0.85)^x$
- (3)  $y = 5200(1.15)^x$
- (4)  $y = 5200(1.85)^x$

- \_\_\_\_\_ 10) Which equation below will produce a decay curve?
  - (1)  $y = 3^{x} 4$ (2)  $y = 2(4)^{x}$ (3)  $y = \frac{1}{2}(3)^{x}$ (4)  $y = 6\left(\frac{1}{3}\right)^{x}$

\_\_\_\_\_11) What is the equation of the relationship below?

(1) $y = 8(3)^{\frac{2}{2}}$	x	-2	0	2	4
(2) $y = 8\left(\frac{1}{2}\right)^{x}$	у	8	24	72	216
(3) $y = 24(3)^x$					
(4) $y = 24(3)^{\frac{x}{2}}$					

12) Match each function with the correct graph:





- 13) Use your knowledge of exponential functions to identify the domain and range.
  - (a) Domain:\_\_\_\_\_

Range:\_\_\_\_\_





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(b) Domain:\_\_\_\_\_

## Review Section:

- 14.) An astronaut drops a rock off the edge of a cliff on the Moon. The distance d(t) in meters, the rock travels after t seconds can be modeled by the function  $d(t) = 0.8t^2$ . What is the average speed, in meters per second, of the rock between 5 and 10 seconds after it was dropped? [1] 12 [3] 60
  - [1] 12 [3] 60 [2] 20 [4] 80

15.) If $A = 3x^2 + 5x - 6$ and $B =$	$-2x^2 - 6x + 7$ , the $A - B$ equals:
$\begin{bmatrix} 1 \end{bmatrix} -5x^2 - 11x + 13$	$[3] -5x^2 - x + 1$
[2] $5x^2 + 11x - 13$	[4] $5x^2 - x + 1$

16.) A functions is shown in the table. If included in the table, which ordered pair, (-4,1) or (1,-4), would result in a relation that is no longer a function? Explain your answer in a complete sentence.

x	f(x)	
4	2	
-1	-4	
0	-2	
3	16	

Name Algebra	Homewo	ork Answer:	5	_	Date _ Expor	nential	Period 9D HW
1.) [2]							
2.) [1]							
3.) [4]							
4.) [2]							
5.) [2]							
6.) [1]							
7.) [1]							
8.) [3]							
9.) [2]							
10.) [4]							
11.) [4]							
12.) (1	– D)	(2 – A)	(3 – F)	(4 – C)	(5 – B)	(6 – E)	
13.) a) b)	Domain { Domain {	$-\infty < x < \infty$ $-\infty < x < \infty$	) Range: { ) Range: {	$1 < y < \infty \}$ $0 < y < \infty \}$			
14.) 1							

- 15.) 2
- 16.) (-4,1) because the x value will repeat which will create a relation.